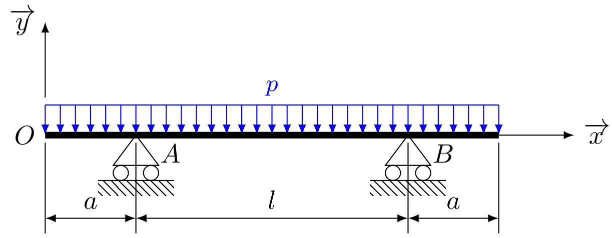
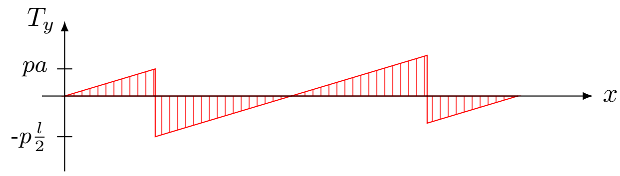
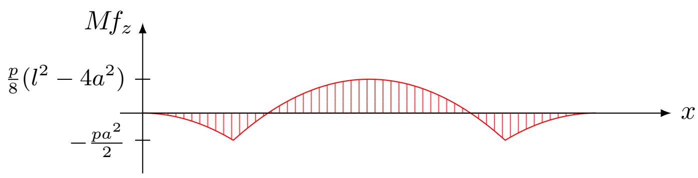
Poutre chargée uniformément sur deux appuis avec porte-à-faux symétrique



Appuis : $R_A = R_B = p(a + \frac{l}{2})$



x	Effort tranchant	Moment fléchissant
0	0	0
a^-	pa	
a^+	$pa - p(a + \frac{l}{2}) = -p\frac{l}{2}$	$-p\frac{a^2}{2}$
$a + \frac{l}{2}$	$-p\frac{l}{2} + p\frac{l}{2} = 0$	$-p\frac{a^2}{2} + p\frac{l}{2}\frac{l}{4} = \frac{p}{8}(l^2 - 4a^2)$
$a + l^-$	$p\frac{l}{2}$	
$a + l^+$	$p\frac{l}{2} - p(a + \frac{l}{2}) = -pa$	$\frac{p}{8}(l^2 - 4a^2) - p\frac{l}{2}\frac{l}{4} = -p\frac{a^2}{2}$
L	$-pa + pa = 0$	$-p\frac{a^2}{2} + p\frac{a^2}{2} = 0$



Déformée :

$$\frac{y''}{EI_{Gz}} = -p\frac{a^2}{2} + p\frac{l}{2}x - p\frac{x^2}{2}$$

$$\frac{y'}{EI_{Gz}} = -p\frac{x^3}{6} + p\frac{l}{4}x^2 - p\frac{a^2}{2}x + c_1$$

$$\frac{y}{EI_{Gz}} = -p\frac{x^4}{24} + p\frac{l}{12}x^3 - p\frac{a^2}{4}x^2 + c_1 x + c_2$$

$$\begin{cases} y'_{E(x=\frac{l}{2})} = 0 \\ y_{A(x=0)} = 0 \end{cases} \quad \begin{cases} -p\frac{l^3}{48} + p\frac{l^3}{16} - p\frac{a^2 l}{4} + c_1 = 0 \\ c_2 = 0 \end{cases}$$

Flèche en $E(x = \frac{l}{2})$

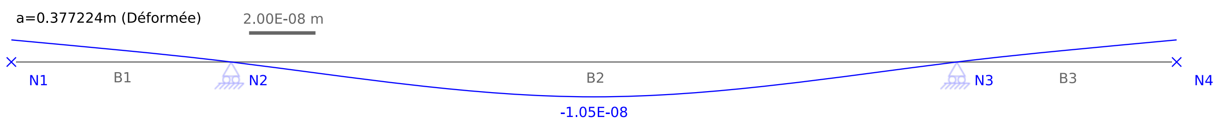
$$\frac{y}{EI_{Gz}} = -p\frac{l^4}{16 \times 24} + p\frac{l^4}{16 \times 6} - p\frac{a^2 l^2}{16} - p\frac{l^4}{16 \times 3} + p\frac{a^2 l^2}{8}$$

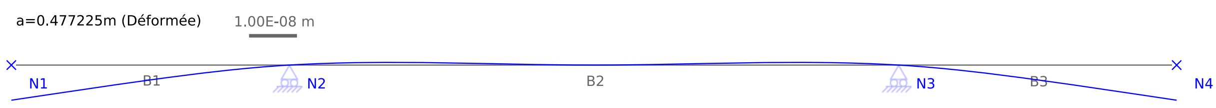
$$= -p\frac{5l^4}{16 \times 24} + p\frac{a^2 l^2}{16}$$

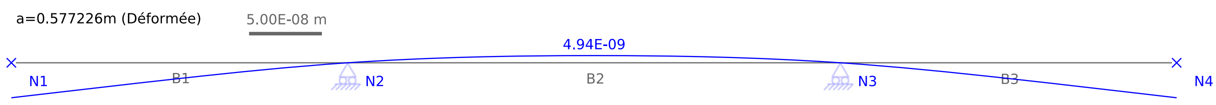
$$\boxed{y_E = -\frac{pl^4}{16EI_{Gz}}\left(\frac{5}{24} - \frac{a^2}{l^2}\right)}$$

$y_E = 0$ pour $L = 2$ et $l = \frac{L}{1+2\sqrt{\frac{5}{24}}} \simeq 1.0455$

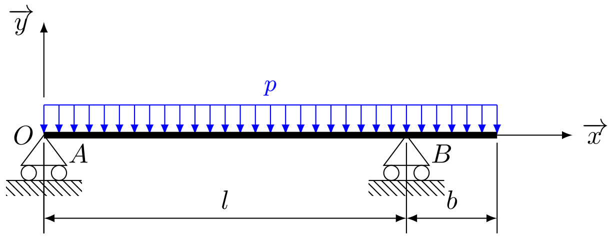
Déformée avec pyBar :







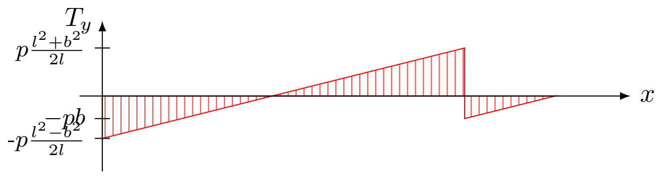
Poutre chargée uniformément sur deux appuis avec porte-à-faux



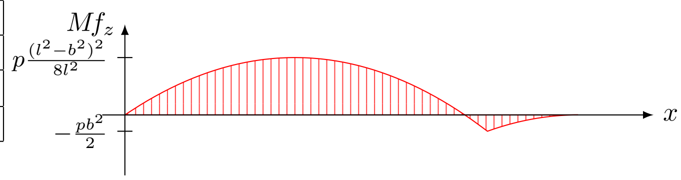
Résultante : $R_{(\frac{l+b}{2})} = p(l+b)$

Appuis :

$$\begin{cases} \sum F = 0 \\ \sum M_A = 0 \end{cases} \qquad \begin{cases} R_A + R_B = p(l+b) \\ R_B l = p(l+b)\frac{l+b}{2} \end{cases} \qquad \begin{cases} R_A = p\frac{l^2-b^2}{2l} \\ R_B = p\frac{(l+b)^2}{2l} \end{cases}$$



x	Effort tranchant	Moment fléchissant
$0-$	0	
$0+$	$-p\frac{l^2-b^2}{2l}$	0
$\frac{l^2-b^2}{2l}$	0	$p\frac{l^2-b^2}{2l}\frac{l^2-b^2}{4l}$
$l-$	$p(l-\frac{l^2-b^2}{2l}) = p\frac{l^2+b^2}{2l}$	
$l+$	$p\frac{l^2+b^2}{2l} - p\frac{(l+b)^2}{2l} = -pb$	$p\frac{l^2-b^2}{2l}\frac{l^2-b^2}{4l} - p\frac{l^2+b^2}{2l}\frac{l^2+b^2}{4l}$
$l+b$	$-pb+pb=0$	$-p\frac{b^2}{2}+p\frac{b^2}{2}=0$



Déformée entre A et B :

$$\frac{y''}{EI_{Gz}} = -p\frac{l^2-b^2}{2l}x - p\frac{x^2}{2}$$

$$\frac{y'}{EI_{Gz}} = -p\frac{x^3}{6} + p\frac{l^2-b^2}{4l}x^2 + c_1$$

$$\frac{y}{EI_{Gz}} = -p\frac{x^4}{24} + p\frac{l^2-b^2}{12l}x^3 + c_1x + c_2$$

$$\begin{cases} y_{A(x=0)} = 0 \\ y_{B(x=l)} = 0 \end{cases} \qquad \begin{cases} c_2 = 0 \\ -p\frac{l^4}{24} + p\frac{l^2-b^2}{12}l^2 + c_1 l = 0 \end{cases}$$

$$\frac{y'_{B(x=l)}}{EI_{Gz}} = -p\frac{l^3}{6} + p\frac{l^2-b^2}{4}l + c_1 = c_3$$

Déformée entre B et C :

$$\frac{y''}{EI_{Gz}} = -p\frac{b^2}{2} + pbx - p\frac{x^2}{2}$$

$$\frac{y'}{EI_{Gz}} = -p\frac{x^3}{6} + p\frac{b}{2}x^2 - p\frac{b^2}{2}x + c_3$$

$$\frac{y}{EI_{Gz}} = -p\frac{x^4}{24} + p\frac{b}{6}x^3 - p\frac{b^2}{4}x^2 + c_3x + c_4$$

$$\begin{cases} y_{B(x=0)} = 0 \\ y'_{B(x=0)} = c_3 = -p\frac{l^3}{6} + p\frac{l^2-b^2}{4}l + p\frac{l^3}{24} - p\frac{l^2-b^2}{12}l = p\frac{l^3}{24} - p\frac{b^2 l}{6} \end{cases}$$

Flèche en $C(x=b)$:

$$\frac{y}{EI_{Gz}} = -p\frac{b^4}{24} + p\frac{b^4}{6} - p\frac{b^4}{4} + c_3 b$$
$$= -p\frac{3b^4}{24} + p\frac{l^3 b}{24} - p\frac{4b^3 l}{24}$$

$$\boxed{y_C = -\frac{pb(l+b)}{24EI_{Gz}}(3b^2 + lb - l^2)}$$

Calcul de b pour $y_C = 0$ et $L = 1.25$:

$$3b^2 + lb - l^2 = 0$$
$$3b^2 + (L-b)b - (L-b)^2 = 0$$
$$b^2 + 3Lb - L^2 = 0$$

$$b = \frac{-3L+\sqrt{13L^2}}{2} = \frac{\sqrt{13}-3}{2}L \simeq 0.3785$$

Déformée avec pyBar :